

# Computational Algorithm for Reconstructing the Profile of 2-D Rough Surfaces

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## Abstract

The presented reconstruction algorithm is based on merging the fast multipole method (FMM) for the forward solver with the rapidly convergent descent method for the cost function minimization algorithm (by Fletcher and Powell 1964 and 1970). Parametric results are presented showing the potential of the proposed computational algorithm.

## I. INTRODUCTION

The presence of rough ground surfaces is considered a major source of clutter in subsurface sensing applications. The ground response can be significantly stronger than the buried target signature. In addition, the ground roughness causes considerable distortion to ground penetrating radar (GPR) received signals, making it fruitless to process the data using conventional signal processing techniques. While it is not usually feasible to experimentally determine the unknown rough ground profile, however, fast reconstruction algorithms can be developed to reconstruct the profile based on collected GPR data. The current work is considered an initial effort to remove ground response from GPR data for target imaging. In real situation, the ground is rough in two dimensions (i.e.  $x$  and  $y$  directions) which makes it more challenging to be reconstructed.

Previous works to reconstruct 1-D and 2-D rough surfaces were based on using the Kirchhoff approximation [1-2], to compute the electromagnetic waves scattered from the rough surface [3]. The focus of the current work will be on developing a fast computational inversion algorithm for reconstructing 2-D dielectric rough surfaces. The algorithm is based on combining a fast forward solver with an efficient searching technique. It is also based on using the electromagnetic waves (GPR-type data) scattered from the rough ground surface to retrieve the surface height variation. For simplicity, synthetic GPR-type data will be used here (i.e. no real data). The previously well developed steepest descent fast multipole method (SDFMM) for targets buried under a 2-D dielectric random rough ground surfaces will be used as the fast forward solver [4]. In addition, the efficient optimization search technique based on the algorithm of Fletcher and Powell, for rapidly convergent descent method in minimizing a cost function, will be employed [5].

Several key issues need to be examined; the effect of the incident and scatter polarization and directions, the location of receivers (e.g. far-zone or near-zone), computational expenses of the algorithm, cost function type, and mathematical model of the rough surface and its unknown parameters.

## II. FORMULATIONS

The cost function represents the mean square error between synthetic data and simulated data of scattered electric fields (GPR-type data). In this work, the cost function  $C(\theta)$  is defined by:

$$C(\theta) = \sum_{i=1}^{N_r} \left| \bar{E}_i^{True} - \bar{E}_i^{Sim} \right|^2 \quad (1)$$

in which,  $\bar{E}_i^{True}$  and  $\bar{E}_i^{Sim}$  represent the scattered electric fields for true-data (GPR-type data) and simulated data, respectively, at receiver number  $i$ . The total number of receivers (sensors) is given by  $N_r$ . All receivers are located above the rough ground. The electric fields in (1) are obtained at single frequency and single polarization of the incident waves. The vector  $\theta$  represents a vector of unknown parameters that need to be recovered in order to reconstruct the rough surface profile. The cost function in (1) will be minimized as follows:

$$\hat{\theta} = \arg_{\theta}(\min(C(\theta))) \quad (2)$$

in which  $\hat{\theta}$  represents the vector of the obtained estimated parameters. The minimization process, i.e. the optimization technique, will be conducted using a rapid and efficient steepest decent approach, this algorithm was developed by Fletcher and Powell [5]. The optimization algorithm involves evaluating the gradient of the cost function with respect to each unknown parameter. This scenario necessitates using a fast forward solver in the inversion algorithm, such as the SDFMM. For faster and more efficient computations, the elements of the unknown vector  $\theta$  will be restricted to certain limits. In other words, upper and lower bound constraints are *a priori* provided to the optimizer, i.e.  $\theta_{LB} \leq \theta \leq \theta_{UB}$ . The iterative inversion technique to search for the unknown parameter vector  $\theta$  is given by [5]:

$$\hat{\theta}_{k+1} = \hat{\theta}_k + \alpha_k d_k \quad (3)$$

in which  $k$  is the iteration index,  $\alpha_k$  is the  $k$ -step, and the vector  $d_k$  is the vector that minimizes the quadratic equation [5].

The geometry of the problem is shown in Fig. 1 and the flowchart of the algorithm is demonstrated in Fig. 2.

## III. NUMERICAL RESULTS

Several key issues of the inversion algorithm will be discussed such as: the mathematical model of the unknown rough surface profile with its unknown parameters, the behavior of cost functions versus these parameters, and the convergence of the algorithm with respect to the initial guess of these parameters. Numerical examples to clarify these issues will be presented here. The results are obtained using the fast forward solver SDFMM for a surface size of



$1.224 \times 1.224 \text{ m}^2$  at  $f = 1 \text{ GHz}$  (free space wave length  $\lambda_0 = 30$ ). The incident electromagnetic wave is represented by a Gaussian beam normally incident to the surface with horizontal polarization [4]. The incident beam illuminates a circular spot on the ground of diameter 40cm, Fig. 1. The relative dielectric constant of the ground is assumed  $\epsilon_r = 2.5 - i0.18$ . The horizontally polarized electric field scattered from the ground in the far-zone at normal incidence is calculated to obtain the cost function  $C(\theta)$ . This implies that the co-polarized waves ( $HH$ ) are obtained at a single receiver (i.e.,  $N_r = 1$  in (1)).

Example 1 focuses on a 2-D sinusoidal rough surface [1]. The model of the surface is  $h(x, y) = H \cos(2\pi x/L) \cos(2\pi y/L)$ , where  $H$  and  $L$  are the surface maximum height and period, respectively. The behavior of the cost function  $C(\theta)$  is plotted versus the surface parameters  $H$  and  $L$  as shown in Fig. 3a. The results show a pronounced local minimum at  $L = 1\lambda_0$  when for large surface heights more than for small heights. This indicates that smoother surface profiles could be more difficult to reconstruct. The inversion algorithm is tested to recover the unknown parameter  $H$ , assuming, for simplicity, that the surface period is known ( $L = 1\lambda_0$ ), as shown in Fig. 3b. Zero initial value of  $H$  (i.e., flat surface) has been used in the algorithm recovering a sinusoidal surface with relative error less than 3% with respect to the true surface.

In Example 2, the previous test is repeated for a 2-D random rough surface modeled by tensor-product B-spline function [6]. The surface model is given by

$$h(x, y) = \sum_{n=1}^{N_n} \sum_{m=1}^{N_m} \alpha_{n,m} S_n(x) S_m(y), \text{ where } \alpha_{n,m} \text{ represents the unknown coefficients, } S_n(x) \text{ and } S_m(y) \text{ are the B-spline functions for } x \text{ and } y, \text{ respectively, } N_n \text{ and } N_m \text{ are the total number of the unknown coefficients in } x \text{ and } y\text{-directions, respectively. The number of coefficients is assumed } N_n = N_m = 16. \text{ For simplicity, 254 coefficients are assumed known, while only two coefficients are assumed unknown. These 254 coefficients are obtained using a uniform random number generator. The generated random rough surface is enforced to have a zero mean height. The inversion algorithm is tested to recover these two unknown parameters, } \alpha_{7,15} \text{ and } \alpha_{11,2}. \text{ Zero initial values of } \alpha_{7,15} \text{ and } \alpha_{11,2} \text{ (i.e., flat surface) are used in the inversion algorithm, recovering the true values of the coefficients as shown in Fig. 4.}$$

The inversion algorithm required 76 and 88 runs of the 3-D SDFMM forward solver to achieve  $10^{-6}$  error in the cost function for the sinusoidal and Tensor-product B-spline surface, respectively. Each run required 231 MB computer memory and approximately 15 CPU minutes, to achieve tolerance of  $10^{-3}$  using the TFQMR iterative solver (transpose-free quasi-minimal residual algorithm). All computational work is conducted using the COMPAQ ALPHA server 667 MHz server.

The inversion algorithm is tested on a groove-like dielectric rough surface (1-D) of dimensions  $1.0 \times 1.0 \text{ m}^2$  [2]. A variety of strategies are demonstrated in the inversion algorithm. These strategies are the multiple-incidence strategy, the multiple-frequency strategy, and/or combination of both strategies. The numerical results of reconstructing the groove-like surface using the multiple-incidence are compared with those using single incidence, as shown in Fig. 5. In this example, we assumed  $\epsilon_r = 4 - i0.01$  and eleven receivers are located at 15 cm above the ground mean plane, and separated by 6cm. The results show that multiple-incidence provides better reconstruction of the surface.

## ACKNOWLEDGMENTS

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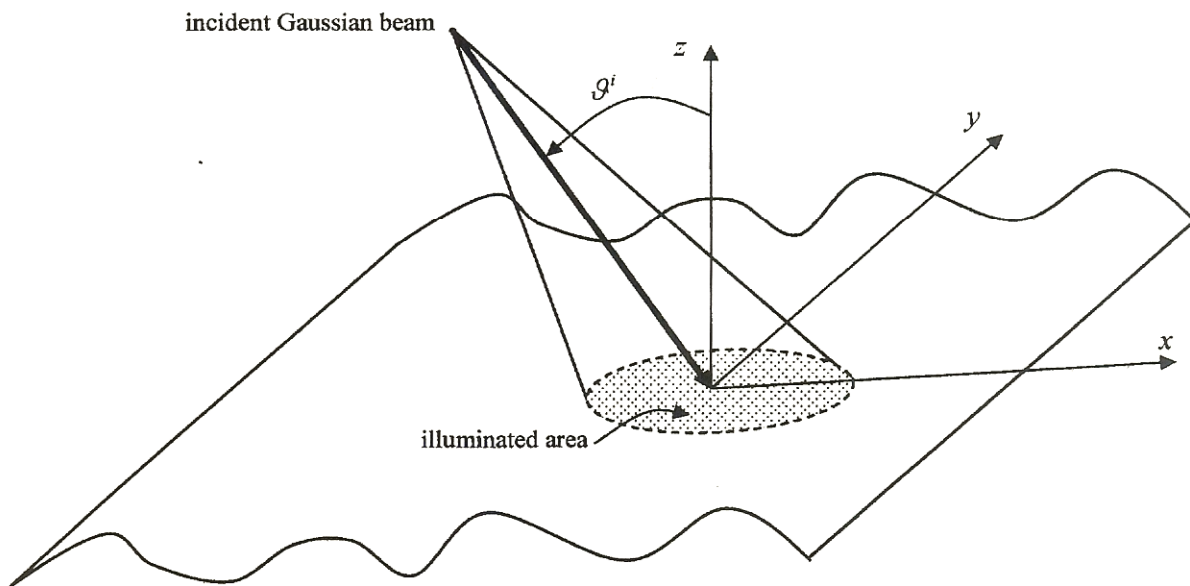


Fig. 1 Groove-like rough surface illuminated in  $x$ - $y$  plane by a 2-D Gaussian beam

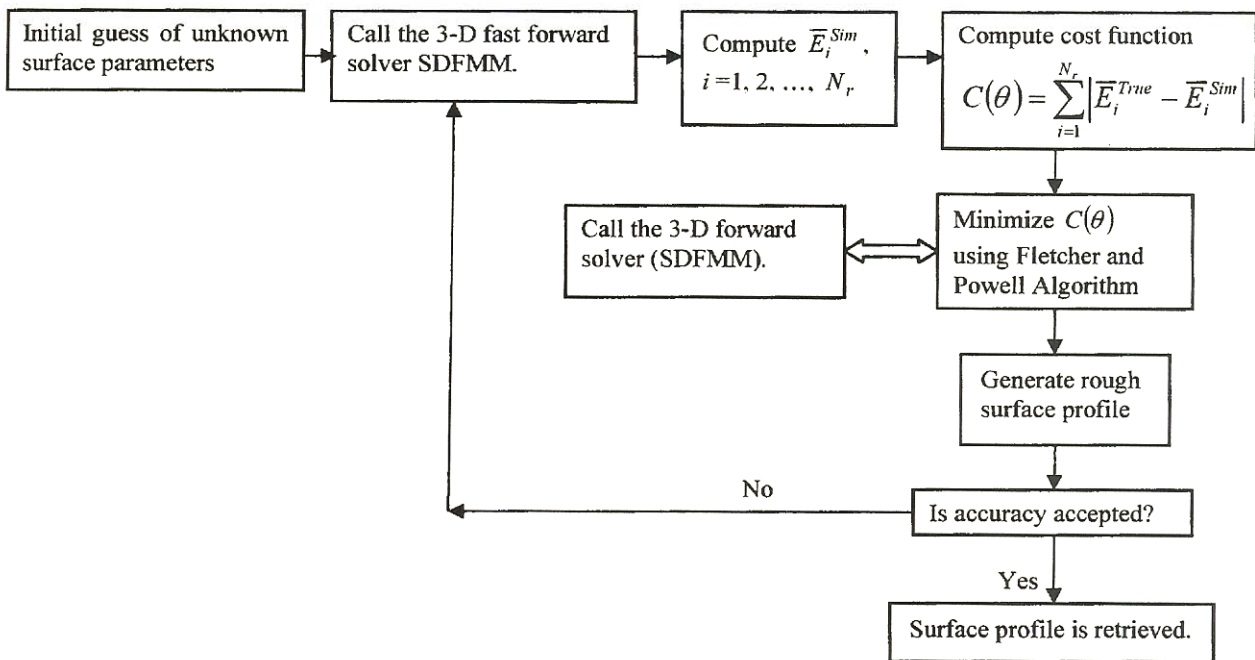


Fig. 2 Flowchart of inversion algorithm

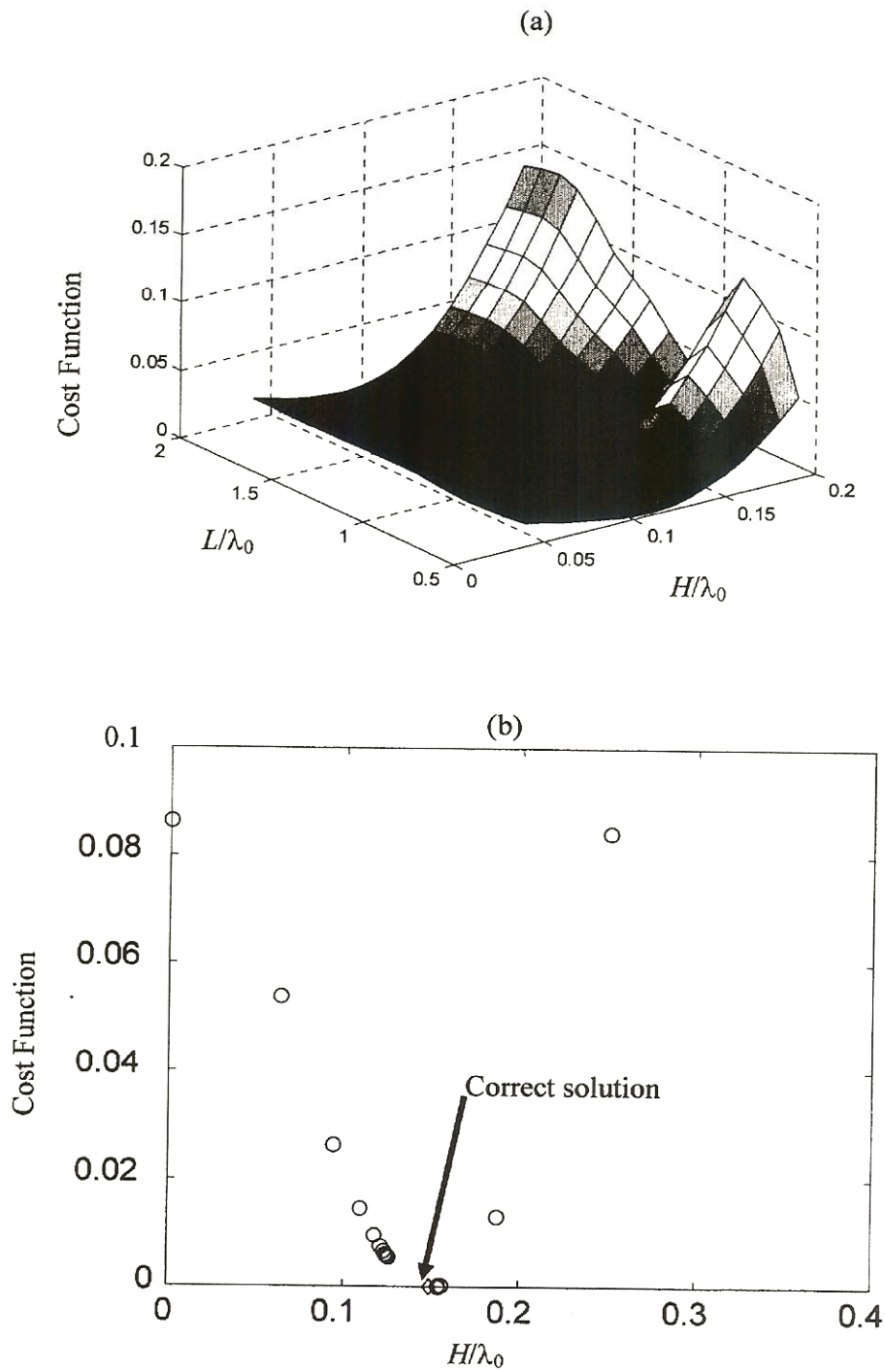


Fig. 3a-b 2-D sinusoidal surface, (a) cost function behavior, (b) convergence of inversion algorithm assuming the surface period  $L = 1\lambda_0$ . True value is represented by the diamond.



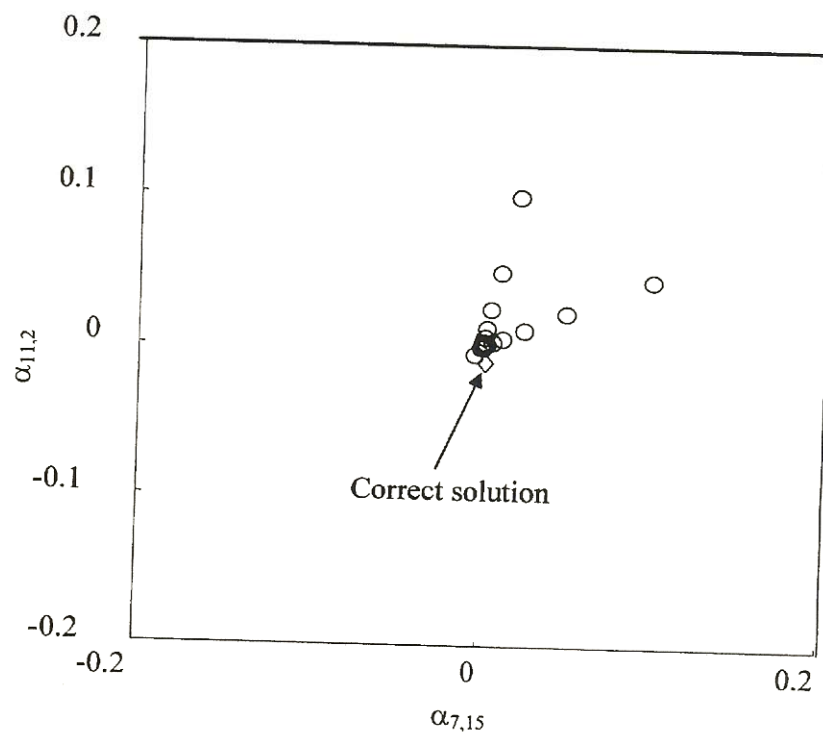


Fig. 4 Convergence of inversion algorithm for 2-D Spline-product surface model with 2 unknown parameters ( $\alpha_{7,15}$  and  $\alpha_{11,2}$ ). True value is represented by the diamond.