Scattering from Dielectric Targets Buried Beneath 2-D Randomly Rough Surfaces

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ABSTRACT

This work presents the Mueller matrix elements for scattering from dielectric targets buried beneath 2-D random rough surfaces (3-D scattering problem). The fully polarimetric scattering matrix *S* is computed for hundreds of computer generated random rough surface realizations and hence the Mueller matrix elements are obtained. The numerical results show that if one relies only on the co- or cross-polarized intensities (i.e. the magnitude of the four elements of the polarimetric scattering matrix *S*); it is very difficult to sense the buried objects. However, investigating all the sixteen Mueller matrix elements greatly helps in detecting these objects.

Keywords: mine detection, buried objects, rough surface scattering, remote sensing, fast multipole method (FMM), SDFMM.

1. INTRODUCTION

The modified Mueller matrix relates the incident with the scattered waves, which is defined in terms of the modified Stokes vector *I* given by [1], [2]:

$$I = \left[|E_v|^2 \quad |E_h|^2 \quad 2\,\mathrm{Re}\left(E_v E_h^*\right) \quad 2\,\mathrm{Im}\left(E_v E_h^*\right) \right]^T / \eta \tag{1}$$

This implies that the Mueller matrix has sixteen complex elements representing all combinations between real and imaginary parts of co- and cross-polarized scattered fields. The co and cross polarized waves are the elements of the polarimetric scattering matrix *S*, i.e. the *vv*, *hh*, *vh* and *hv*. The polarization of the incident or scattered waves is denoted by *h* or *v*, for horizontal or vertical polarization, respectively, while η is the intrinsic impedance of the surrounding medium.

The three-dimensional scattering solver, the Steepest Descent Fast Multipole Method (SDFMM), is used here to calculate the equivalent electric and magnetic surface currents on the

Detection and Remediation Technologies for Mines and Minelike Targets VIII, Russell S. Harmon, John H. Holloway, Jr., J. T. Broach, Editors, Proceedings of SPIE Vol. 5089 (2003) © 2003 SPIE · 0277-786X/03/$15.00

random rough ground interface and the buried targets [3], [4]. Using the SDFMM made it possible to compute hundreds of Monte Carlo simulations for scattering from dielectric objects buried under the random rough ground. The statistical average of each element in the Mueller matrix is computed for two cases: (i) rough ground alone, and (ii) rough ground with buried targets. In the current work, the statistical average of the scattered electric fields from two buried objects is calculated in the far-field zone. It is important to emphasize that the subtraction process often used to obtain the target signature is not used in the current work. In other words, the far-fields scattered from the rough ground with the buried objects are directly compared with those scattered from the rough ground with no buried objects. From [2], we get:

$$I^s = \frac{1}{r^2} M_m I^i \tag{2}$$

where, the superscripts s and i represent the scattered and incident waves, respectively. The modified Mueller matrix M_m is given by [2]:

$$M_m = \begin{bmatrix} |S_{vv}|^2 & |S_{vh}|^2 & \mathrm{Re}(S_{vh}^* S_{vv}) & -\mathrm{Im}(S_{vh}^* S_{vv}) \\ |S_{hv}|^2 & |S_{hh}|^2 & \mathrm{Re}(S_{hv} S_{hh}^*) & -\mathrm{Im}(S_{hv} S_{hh}^*) \\ 2\,\mathrm{Re}(S_{vv} S_{hv}^*) & 2\,\mathrm{Re}(S_{vh} S_{hh}^*) & \mathrm{Re}(S_{vv} S_{hh}^* + S_{vh} S_{hv}^*) & -\mathrm{Im}(S_{vv} S_{hh}^* - S_{vh} S_{hv}^*) \\ 2\,\mathrm{Im}(S_{vv} S_{hv}^*) & 2\,\mathrm{Im}(S_{vh} S_{hh}^*) & \mathrm{Im}(S_{vv} S_{hh}^* + S_{vh} S_{hv}^*) & \mathrm{Re}(S_{vv} S_{hh}^* - S_{vh} S_{hv}^*) \end{bmatrix} \tag{3}$$

where S_{pq} is the p-polarized scattered waves due to the q-polarized incident waves (p, q = v, h) as described in the fully polarimetric scattering matrix S [2]:

$$\begin{bmatrix} E_v^s \\ E_h^s \end{bmatrix} = \frac{e^{jkr}}{r} \begin{bmatrix} S_{vv} & S_{vh} \\ S_{hv} & S_{hh} \end{bmatrix} \begin{bmatrix} E_v^i \\ E_h^i \end{bmatrix} \tag{4}$$

where $E_{v,h}^i$ and $E_{v,h}^s$ are the vertically and horizontally polarized incident and scattered electric fields in the far-zone, respectively.

2. NUMERICAL RESULTS

This Section presents numerical results for the 16 Mueller matrix elements given in eq. (3). These elements represents scattering from dielectric objects buried beneath the 2-D random rough surface (see Fig. 1). The rough surface is characterized by Gaussian statistics. The dimensions of the ground are assumed $8\lambda_0 \times 8\lambda_0$ with Gaussian half-beam width equal to $1.6\lambda_0$ centered on the ground at $4\lambda_0 \times 4\lambda_0$, where λ_0 is the free space wavelength [3]. For the buried objects, the first one is an oblate spheroid with ($a = c = 0.15\lambda_0, b = 0.3\lambda_0$), the second object is a cylinder with ($a = 0.15\lambda_0, h = 0.9\lambda_0$) 30°-tilted with respect to the edge of the ground.

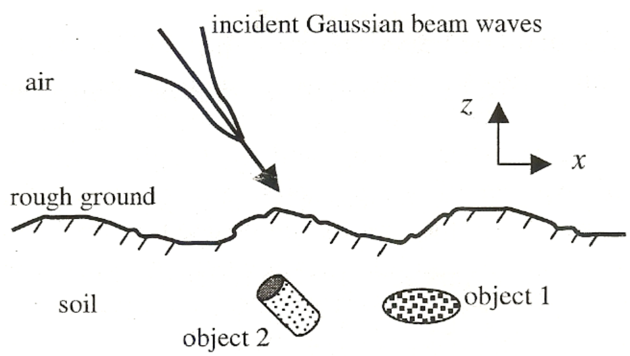

Figure 1. Cross section of a general 2-D rough ground with two buried objects.

The spheroid's center is located at $x = 4.5\lambda_0$, $y = 3.5\lambda_0$ and $z = -0.4\lambda_0$. The horizontal cylinder is tilted by 30° with the x-axis and is located at $x = 4.01\lambda_0$, $y = 4.375\lambda_0$ and $z = -0.4\lambda_0$ measured from the axis mid-point [4]. The rms height and the correlation length of the rough ground are assumed as $\sigma = 0.1\lambda_0$ and $l_c = 0.5\lambda_0$, respectively. The relative dielectric constants of the ground, the cylinder, and the spheroid are assumed as $\varepsilon_r = 2.5 - j0.18$, $\varepsilon_r = 4.0$, and $\varepsilon_r = 2.9 - j0.072$, respectively, at 1 GHz [3]. In this paper, the results are for the statistical average of 100 Monte Carlo simulations are presented. Results for individual rough surfaces can be found in [4].

The 16 Mueller matrix elements given in eq. (3) are plotted for the rough ground alone and the rough ground with buried objects. These elements are related to the co- and cross polarized fields, e.g. $m_{11} = |S_{vv}|^2$, $m_{22} = |S_{hh}|^2$, $m_{14} = -\mathrm{Im}(S_{vh}^* S_{vv})$, etc. All elements presented here are normalized by the factor $4\pi/(2\eta_o A_z P^i)$, where A_Z, η_o, and P^i are the footprint area on the ground, the intrinsic impedance of the free space and the total incident power, respectively, [4]. Figures 2-5 present the 16 elements of the Mueller matrix [4]. The results of m_{14}, m_{24}, m_{41}, and m_{42} clearly show distinguishable difference between the two cases; rough ground alone and rough ground with two buried objects.

3. CONCLUSIONS

This work presented the average of all 16 Mueller matrix elements for scattering from dielectric objects buried beneath the random rough ground. The statistical average for each element is based on the Monte Carlo simulations using the 3-D SDFMM computer code. The numerical results consistently show that investigating all the sixteen Mueller matrix elements significantly help in detecting the buried objects.

ACKNOWLEDGMENTS

This research was sponsored in part by the Northeastern University NSF-ERC award number EEC-9986821 and in part by the Arkansas Science and Technology Authority Grant No AR/ASTA/01-B-18.

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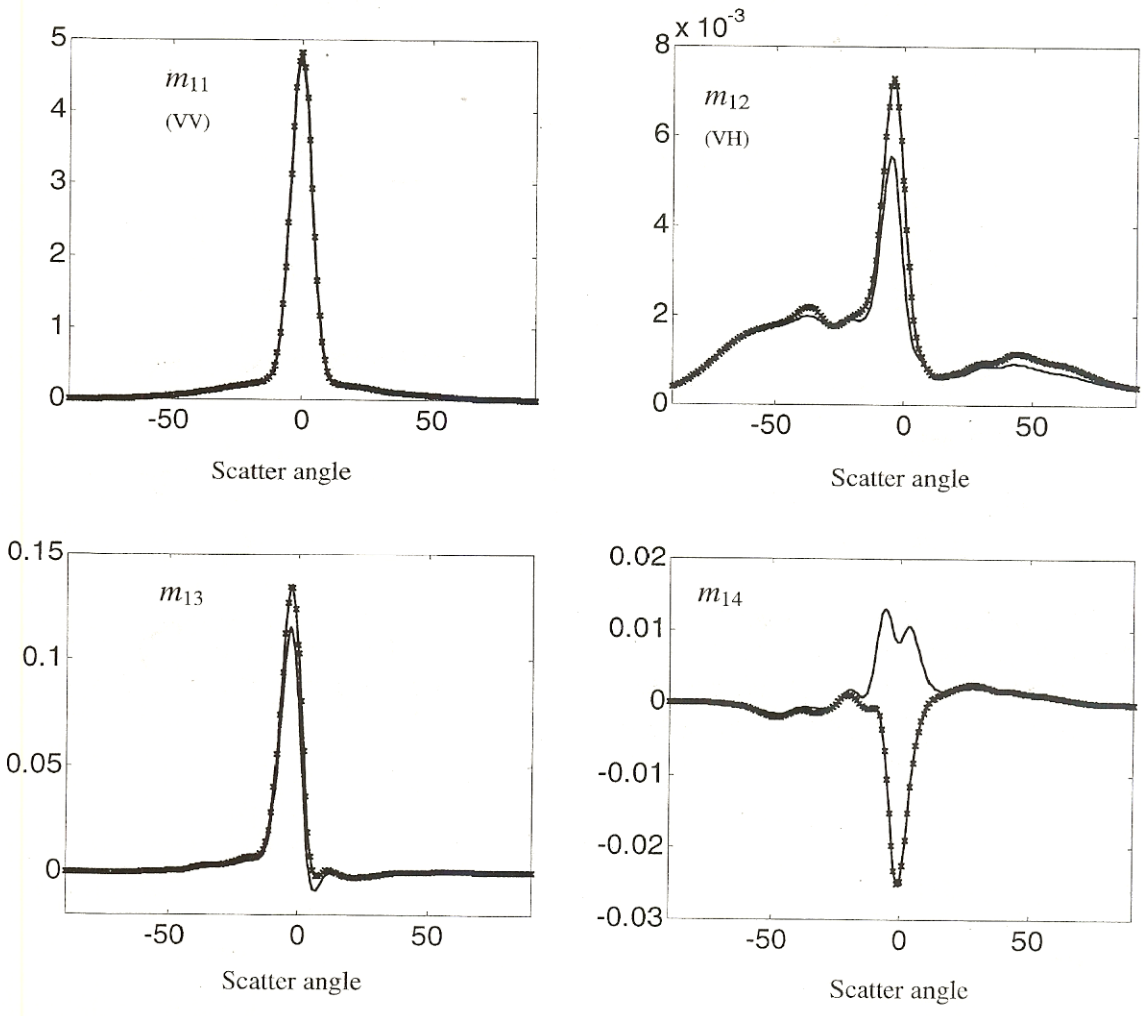

Figure 2a-d. The normalized bistatic modified Mueller matrix elements (total intensity); m_{11}, m_{12}, m_{13}, and m_{14},; the solid line is for the rough ground only and the cross-symbol is for the rough ground with the two buried objects (the spheroid and the horizontal 30°-tilted cylinder). The incident angles are $\theta^i = 0^\circ$ and $\phi^i = 0^\circ$. Monte Carlo for 100 rough surface realizations.

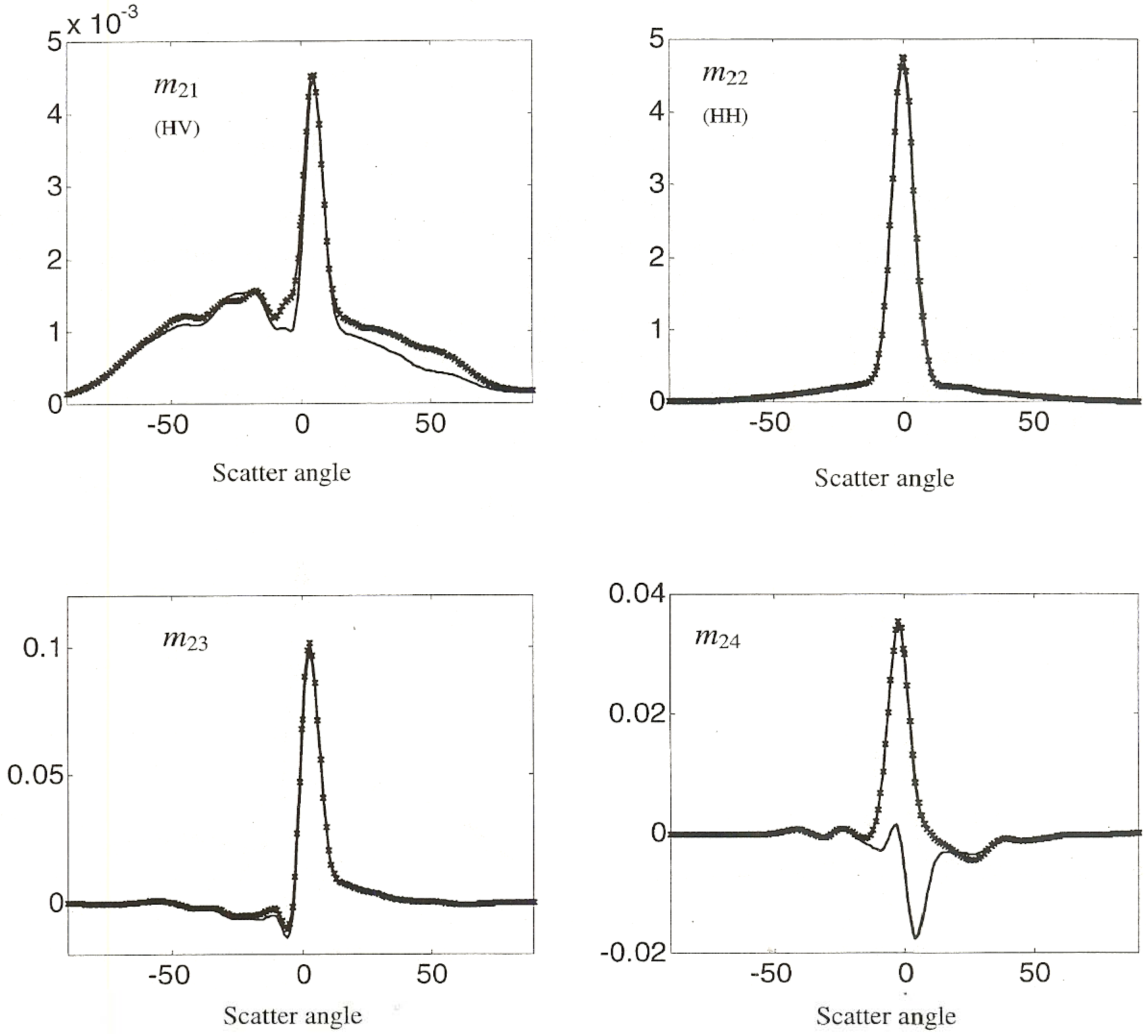

Figure 3a-d. The normalized bistatic modified Mueller matrix elements (total intensity); m_{21}, m_{22}, m_{23}, and m_{24},; the solid line is for the rough ground only and the cross-symbol is for the rough ground with the two buried objects (the spheroid and the horizontal 30°-tilted cylinder). The incident angles are $\theta^i = 0^\circ$ and $\phi^i = 0^\circ$. Monte Carlo for 100 rough surface realizations.

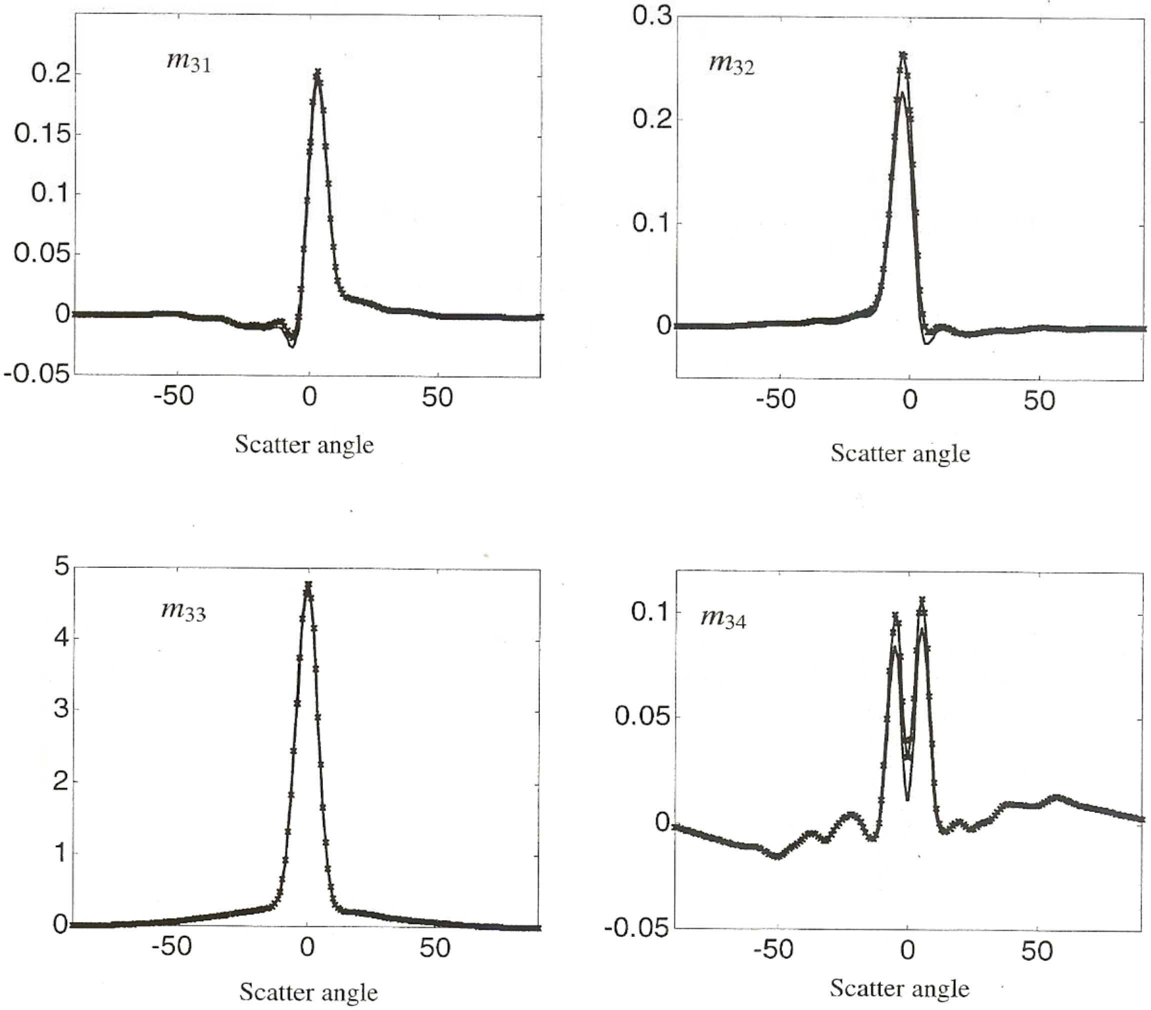

Figure 4a-d. The normalized bistatic modified Mueller matrix elements (total intensity); m_{31}, m_{32}, m_{33}, and m_{34},; the solid line is for the rough ground only and the cross-symbol is for the rough ground with the two buried objects (the spheroid and the horizontal 30°-tilted cylinder). The incident angles are $\theta^i = 0^\circ$ and $\phi^i = 0^\circ$. Monte Carlo for 100 rough surface realizations.

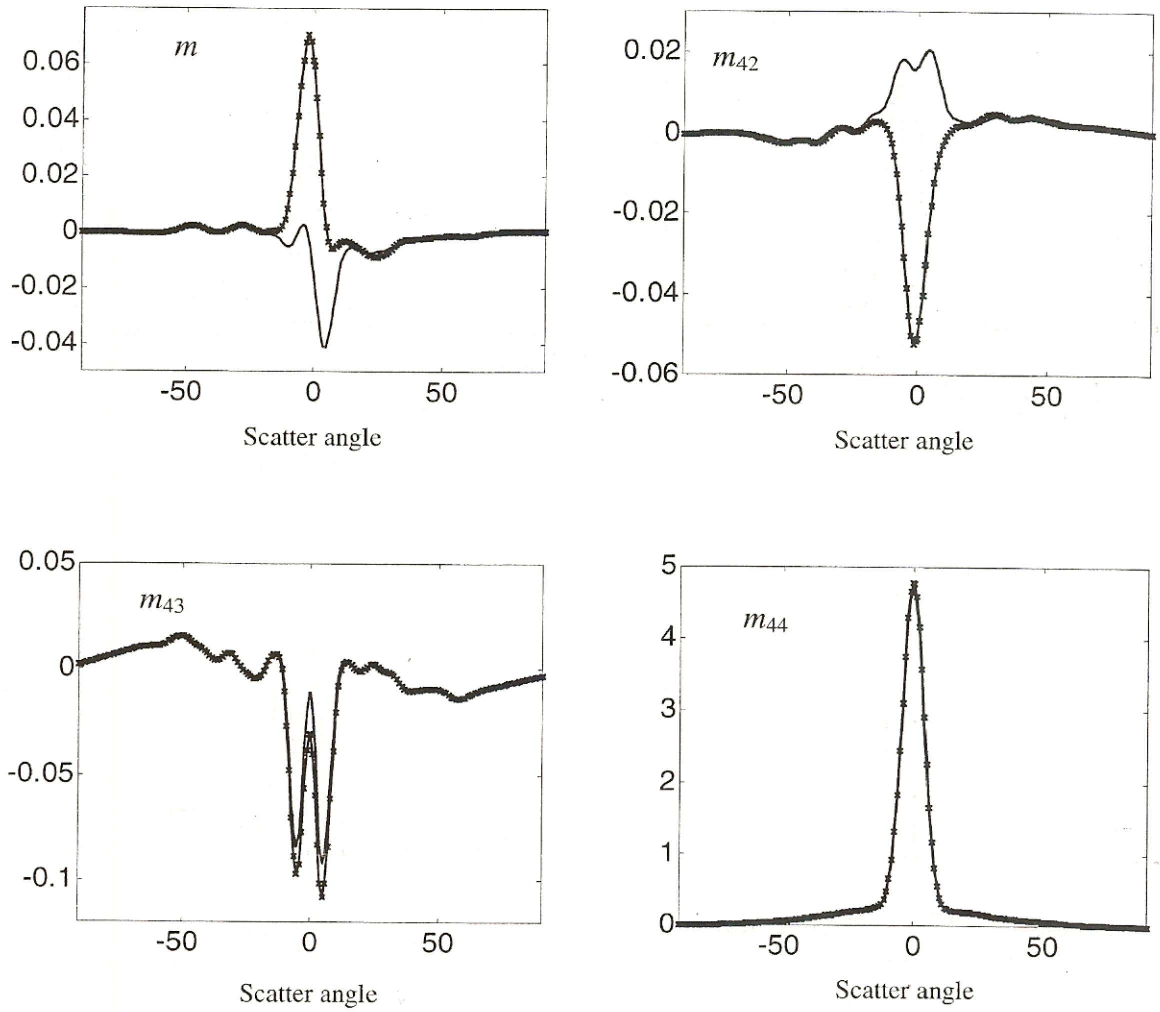

Figure 5a-d. The normalized bistatic modified Mueller matrix elements (total intensity); m_{41}, m_{42}, m_{43}, and m_{44},; the solid line is for the rough ground only and the cross-symbol is for the rough ground with the two buried objects (the spheroid and the horizontal 30°-tilted cylinder). The incident angles are $\theta^i = 0^\circ$ and $\phi^i = 0^\circ$. Monte Carlo for 100 rough surface realizations.