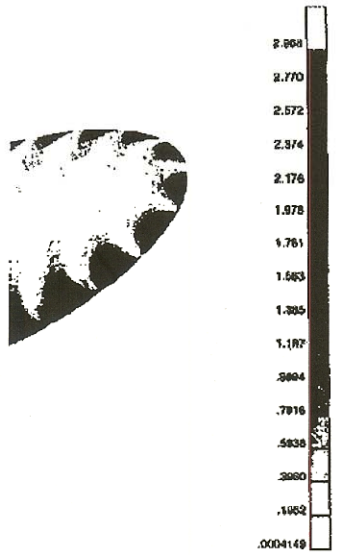

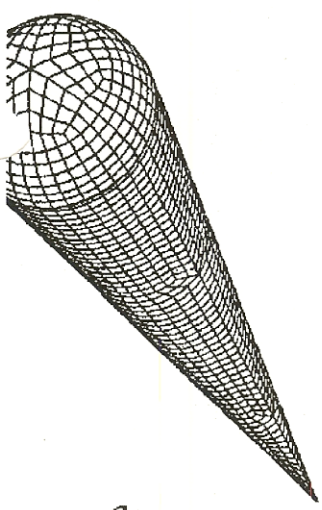

9.

DOUBLE SCATTER RADAR CROSS SECTIONS FOR TWO DIMENSIONAL RANDOM ROUGH SURFACES THAT EXHIBIT BACKSCATTER ENHANCEMENT

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In this work, full wave solutions for the single and double scatter radar cross sections from two dimensional random rough surfaces are given. The solutions are expressed as multidimensional integrals. The high frequency approximations are used to reduce the double scatter integrals from twelve to four dimensional. The single scatter cross section is given in closed form. The large radius of curvature approximation is used. The incident waves are assumed to be plane waves.

Similar to scattering from one dimensional random rough surfaces, the major contributions to the double scatter cross sections come from two different pairs of paths. They are the quasi parallel double scatter paths and the quasi antiparallel double scatter paths. The total incoherent double scatter cross section is the sum of the incoherent quasi parallel and quasi antiparallel double scatter cross sections.

In the high frequency limit, the major contributions to the double and single scatter cross sections come from the neighborhood of the specular points on the rough surface. Thus the surface element scattering coefficients are evaluated at the specular points after integrating with respect to the slopes. The probability density functions of the slopes are assumed to be Gaussian. Shadow functions are included in the expressions.

The effects of changing the rough surface parameters, such as mean square height and mean square slope, on the double scatter cross sections are studied. The level and width of the peak in the backscatter direction depend on the mean square height and slope of the rough surface. The numerical results show sharp enhancement in the backscatter directions. This sharp backscatter enhancement, which is observed for all polarizations and for both normal and oblique incident angles, is associated with the quasi antiparallel double scatter paths.

I. FORMULATION OF THE PROBLEM

The full wave solution for the double scatter far fields $G_d^f(\bar{r})$ from two dimensional rough surfaces $(y = h(x_s, z_s))$ is given by [1]

$$G_d^f(\bar{r}) = \left(\frac{k_0}{2\pi j}\right)^3 \frac{\exp(-jk_0 r)}{r} \int \frac{D_{2'}(\bar{n}^f, \bar{n}')}{\left(n_y^f - n_y'\right)} \exp\left(jk_0 \bar{n}^f \cdot \bar{r}_{s2'}\right) \exp\left(jk_0 \bar{n}' \cdot (\bar{r}_{s2'} - \bar{r}_{s1'})\right)$$
$$\times \frac{D_{1'}(\bar{n}', \bar{n}^i)}{(-n^i + n_y')} \exp\left(-jk_0 \bar{n}^i \cdot \bar{r}_{s1'}\right) \frac{dn_y' dn_x'}{\sqrt{1 - n_y'^2 n_z'^2}} U(\bar{r}_{s1'}) U(\bar{r}_{s2'}) dx_{s1'} dz_{s1'} dx_{s2'} dz_{s2'} G^i(0) \tag{1}$$

in which the time harmonic excitations $\exp(j\omega t)$ are assumed and the free space wave number is $k_0 = \omega\sqrt{\varepsilon_0 \mu_0}$. The incident waves are in the direction $\bar{n}^i$ and the scattered waves are in the direction $\bar{n}^f$ to receive at $\bar{r}$, where

$$\bar{n}^i = n_x^i \bar{e}_x + n_y^i \bar{e}_y + n_z^i \bar{e}_z \tag{2a}$$

$$\bar{n}^f = n_x^f \bar{e}_x + n_y^f \bar{e}_y + n_z^f \bar{e}_z \tag{2b}$$

$$\bar{r} = x\bar{e}_x + y\bar{e}_y + z\bar{e}_z \tag{2c}$$

The scattering matrices at points $1'$ and $2'$ on the rough surface are $D_{1'}(\bar{n}', \bar{n}^i)$ and $D_{2'}(\bar{n}^f, \bar{n}')$. The elements of the scattering matrices depend on the local slope of the rough surface [1]. Moreover, they depend on the polarization of the incident and scattered waves and the media on both sides of the rough interface. The incident fields are assumed to be plane waves and the receiver is located in the far field. The wavevectors of the scattered waves at the point on the surface at $\bar{r}_{s1'}$ are in the direction $\bar{n}' = n'_x\bar{e}_x + n'_y\bar{e}_y + n'_z\bar{e}_z$ (see Fig. 1). The position vectors to points $1'$ and $2'$ on the rough surface (see Fig. 1) are given by

$$\bar{r}_{s1'} = x_{s1'}\bar{e}_x + h(x_{s1'}, z_{s1'})\bar{e}_y + z_{s1'}\bar{e}_z \tag{3a}$$

$$\bar{r}_{s2'} = x_{s2'}\bar{e}_x + h(x_{s2'}, z_{s2'})\bar{e}_y + z_{s2'}\bar{e}_z \tag{3b}$$

At high frequencies, the shadow functions $U(\bar{r}_{s1'})$ and $U(\bar{r}_{s2'})$ are equal to one if the point at $\bar{r}_{s1'}$ is illuminated by the incident waves and visible at point $2'$ on the surface and if the point at $\bar{r}_{s2'}$ is illuminated by a point source at $1'$ and visible at the receiver [2]. The double scatter average cross section is obtained by multiplying (1) by its complex conjugate. The radar cross section for the two dimensional rough surface is defined as

$$\sigma T = \frac{4\pi^2}{A}\left|\frac{G^f}{G^i}\right|^2 \tag{4}$$

in which A is the radar footprint, thus

$$\begin{aligned}
\sigma T = \frac{k_0^6}{16A\pi^5}\int \frac{D_{2'}(\bar{n}^f, \bar{n}')\, D^*_{2''}(\bar{n}^f, \bar{n}'')\, D_{1'}(\bar{n}', \bar{n}^i)\, D^*_{1''}(\bar{n}'', \bar{n}^i)}{\left(n^f_y - n'_y\right)\left(n^f_y - n''_y\right)\left(-n^i_y + n'_y\right)\left(-n^i_y + n''_y\right)} \\
\times \exp\left\{jk_0\left[n^f_x\left(x'_{s2} - x''_{s2}\right) + n^f_y(h_{2'} - h_{2''}) + n^f_z(z_{s2'} - z_{s2''})\right]\right\} \\
\times \exp\left\{-jk_0\left[n'_x(x_{s2'} - x_{s1'}) + n'_y(h_{2'} - h_{1'}) + n'_z(z_{s2'} - z_{s1'})\right]\right\} \\
\times \exp\left\{jk_0\left[n''_x(x_{s2''} - x_{s1''}) + n''_y(h_{2''} - h_{1''}) + n''_z(z_{s2''} - z_{s1''})\right]\right\} \\
\times \exp\left\{-jk_0\left[n^i_x(x_{s1'} - x_{s1''}) + n^i_y(h_{1'} - h_{1''}) + n^i_z(z_{s1'} - z_{s1''})\right]\right\} \\
\times \frac{dn'_y dn'_z}{\sqrt{1 - {n'_y}^2 - {n'_z}^2}}\,\frac{dn''_y dn''_z}{\sqrt{1 - {n''_y}^2 - {n''_z}^2}}\, dx_{s1'}dz_{s1'}dx_{s2'}dz_{s2'}dx_{s1''}dz_{s1''}dx_{s2''}dz_{s2''}
\end{aligned} \tag{5}$$

For the quasi parallel double scatter paths ($n'_\alpha, n''_\alpha < 0$ and $n'_\alpha, n''_\alpha > 0$; $\alpha = x, z$), the following transformations of variables are used:

$$x_{d1} = x_{s1'} - x_{s1''}, \quad x_{d2} = x_{s2'} - x_{s2''}, \tag{6a}$$

$$z_{d1} = z_{s1'} - z_{s1''}, \quad z_{d2} = z_{s2'} - z_{s2''} \tag{6b}$$

and

$$x_{a1} = (x_{s1'} + x_{s1''})/2, \qquad x_{a2} = (x_{s2'} + x_{s2''})/2 \tag{7a}$$

$$z_{a1} = (z_{s1'} + z_{s1''})/2, \qquad z_{a2} = (z_{s2'} + z_{s2''})/2 \tag{7b}$$

Thus from (5), (6) and (7), one gets for the quasi parallel case

$$\sigma T = \frac{k_0^6}{16A\pi^3} \int \frac{D_{2'}(\bar{n}^f, \bar{n}') D^s_{2''}(\bar{n}^f, \bar{n}'') D_{1'}(\bar{n}', \bar{n}^i) D^s_{1''}(\bar{n}'', \bar{n}^i)}{\left(n_y^f - n_y'\right)\left(n_y^f - n_y''\right)\left(-n_y^i + n_y'\right)\left(-n_y^i + n_y''\right)}$$

$$\times \exp\left\{jk_0\left[n_x^f x_{d2} - n_x'(0.5x_{d2} - 0.5x_{d1} + x_{a2} - x_{a1}) + n_x''(-0.5x_{d2} + 0.5x_{d1} + x_{a2} - x_{a1}) - n_x^i x_{d1}\right]\right\}$$

$$\times \exp\left\{jk_0\left[n_z^f z_{d2} - n_z'(0.5z_{d2} - 0.5z_{d1} + z_{a2} - z_{a1}) + n_z''(-0.5z_{d2} + 0.5z_{d1} + z_{a2} - z_{a1}) - n_z^i z_{d1}\right]\right\}$$

$$\times \exp\left\{jk_0\left[n_y^f(h_{2'} - h_{2''}) - n_y'(h_{2'} - h_{1'}) + n_y''(h_{2''} - h_{1''}) - n_y^i(h_{1'} - h_{1''})\right]\right\}$$

$$\times \frac{dn_y' dn_z'}{\sqrt{1 - {n_y'}^2 - {n_z'}^2}} \frac{dn_y'' dn_z''}{\sqrt{1 - {n_y''}^2 - {n_z''}^2}} dx_{d1} dz_{d1} dx_{d2} dz_{d2} dx_{a1} dz_{a1} dx_{a2} dz_{a2} \tag{8}$$

The major contributions to the double and single scatter cross sections, in the high frequency limit, come from the neighborhood of the specular points of the rough surface. The heights at any two neighbor points on the two dimensional rough surface are expanded and written as functions of the heights and slopes at the midpoint between them. The heights at points $1'$ and $1''$ on the rough surface are expanded about the heights at the midpoint between them. The statistical average of the radar cross section (8) with respect to the random heights and slopes of the surface is taken. Upon changing the integration variables n_y', n_y'', n_z', and n_z'' to the spherical coordinate variables ϑ' and ϕ' one gets

$$\left\langle \sigma_{dp}^{PQ} \right\rangle = \frac{(2k_0 L_m)^2}{\pi} P_2(\bar{n}^i) P_2(\bar{n}^f) \sum_{R,S=V,H} \left\{ \int \frac{\left[D^{PS}_{2'}(\bar{n}^f, \bar{n}') D^{SQ}_{1'}(\bar{n}', \bar{n}^i) D^{*PR}_{2''}(\bar{n}^f, \bar{n}'') D^{*RQ}_{1''}(\bar{n}'', \bar{n}^i)\right]}{\left(n_y^f - n_y'\right)\left(n_y^f - n_y''\right)\left(-n_y^i + n_y'\right)\left(-n_y^i + n_y''\right)} \right.$$

$$\times \frac{p(h_{xc1s}, h_{xc2s}, h_{zc1s}, h_{zc2s})}{\left(n_y^f - (n_y' + n_y'')/2\right)^2 \left(-n_y^i + (n_y' + n_y'')/2\right)^2}$$

$$\left[1 - P_2(|n_y'|)\right]\left[1 - P_2(|n_y''|)\right] \operatorname{sinc}\left[k_0 L_m(n_x' - n_x'')\right] \operatorname{sinc}\left[k_0 L_m(n_z' - n_z'')\right]$$

$$\left. \times \exp\left(-\langle h^2 \rangle k_0^2 (n_y'' - n_y')^2\right) \sin\vartheta' \sin\vartheta'' d\vartheta' d\vartheta'' d\phi' d\phi'' \right\} \tag{9}$$

The integrations with respect to x_{d1}, x_{d2}, z_{d1}, and z_{d2}, yield Dirac delta functions in the high frequency limit. In (9) the surface element scattering coefficients D^{PQ} are evaluated at the slopes at the specular points h_{xc1s}, h_{xc2s}, h_{zc1s} and h_{zc2s}, as a result of integrating the Dirac Delta functions $\delta(.)$ with respect to the random slopes. The probability density functions of the (large scale) slopes $p(h_{xc1}, h_{xc2}, h_{zc1}, h_{zc2})$ are assumed to be Gaussian. The slopes at the specular points are given by

$$h_{xc1s} = -\left[-n_x^i + \frac{n_x' + n_x''}{2}\right] \Big/ \left[-n_y^i + \frac{n_y' + n_y''}{2}\right] \tag{10a}$$

$$h_{xc2s} = -\left[-n_x^f + \frac{n_x' + n_x''}{2}\right] \Big/ \left[-n_y^f + \frac{n_y' + n_y''}{2}\right] \tag{10b}$$

$$h_{zc1s} = -\left[-n_z^i + \frac{n_z' + n_z''}{2}\right] \Big/ \left[-n_y^i + \frac{n_y' + n_y''}{2}\right] \tag{10c}$$

$$h_{xc2s} = -\left[-n_x^f + \frac{n_x' + n_x''}{2}\right] / \left[-n_y^f + \frac{n_y' + n_y''}{2}\right] \tag{10d}$$

The integrations over x_{s1}, x_{s2}, z_{s1} and z_{s2} yield the footprint area A and the sinc functions. Furthermore, L_m is the mean width of a typical depression on the rough surface [1], [3], [4]. The probabilities that the surface does not shadow the incident and scattered waves are given by $P_2(\hat{n}^i)$ and $P_2(\hat{n}^f)$, respectively [2], and $[1 - P_2]$ is associated with the probability of a double scatter event. The symbols V and H are for vertical and horizontal polarizations. For the quasi antiparallel double scatter paths, the following transformations of variables are used in (5):

$$x_{d1} = x_{s1'} - x_{s2''}, \qquad x_{d2} = x_{s2'} - x_{s1''}, \tag{11a}$$

$$z_{d1} = z_{s1'} - z_{s2''}, \qquad z_{d2} = z_{s2'} - z_{s1''} \tag{11b}$$

Thus, one gets the following expression for the high frequency quasi antiparallel ($n_\sigma' < 0$, $n_\sigma'' > 0$ and $n_\sigma' > 0$, $n_\sigma'' < 0$; $\sigma = x, z$) double scatter cross section

$$\left\langle \sigma_{dp}^{PQ} \right\rangle = \frac{(2k_0 Lm)^2}{\pi} P_2(\hat{n}^i) P_2(\hat{n}^f) \sum_{R,S=V,H} \left\{ \int \frac{\left[D_{2'}^{PS}(\hat{n}^f, \hat{n}') D_{1'}^{SQ}(\hat{n}', \hat{n}^i) D_{2''}^{*PR}(\hat{n}^f, \hat{n}'') D_{1''}^{*RQ}(\hat{n}'', \hat{n}^i)\right]}{\left(n_y^f - n_y'\right)\left(n_y^f - n_y''\right)\left(-n_y^i + n_y'\right)\left(-n_y^i + n_y''\right)} \right.$$

$$\times \frac{p(h_{xc1s}, h_{xc2s}, h_{zc1s}, h_{zc2s})}{\left(\left(n_x^f + n_x' - n_x'' - n_x^i\right)/2\right)^2 \left(\left(n_z^f - n_z' + n_z'' - n_z^i\right)/2\right)^2}$$

$$[1 - P_2(|n_x'|)][1 - P_2(|n_z''|)] \operatorname{sinc}\left[k_0 L_m (n_x^f + n_x^i - n_x' - n_x'')\right] \operatorname{sinc}\left[k_0 L_m (n_z^f + n_z^i - n_z' - n_z'')\right]$$

$$\left. \times \exp\left(-\langle h^2 \rangle k_0^2 (n_y^f - n_y' - n_y'' + n_y^i)^2\right) \sin\vartheta' \sin\vartheta'' d\vartheta' d\vartheta'' d\varphi' d\varphi'' \right\} \tag{12}$$

The slopes at the specular points for the quasi antiparallel case are given by

$$h_{xc1s} = -\left[-n_x^i + n_x' - n_x'' + n_x^f\right] / \left[n_y^f - n_y^i + n_y' - n_y''\right] \tag{13a}$$

$$h_{xc2s} = -\left[n_x^f - n_x' + n_x'' - n_x^i\right] / \left[n_y^f - n_y' + n_y'' - n_y^i\right] \tag{13b}$$

$$h_{zc1s} = -\left[n_z^f - n_z^i + n_z' - n_z''\right] / \left[n_y^f - n_y^i + n_y' - n_y''\right] \tag{13c}$$

$$h_{zc2s} = -\left[n_z^f - n_z' + n_z'' - n_z^i\right] / \left[n_y^f - n_y' + n_y'' - n_y^i\right] \tag{13d}$$

The sharp enhancement in the backscatter direction $(-\hat{n}^i = \hat{n}^f)$ is associated with the quasi antiparallel $(\hat{n}' \approx -\hat{n}'')$ double scatter cross section (12). Note the difference in the expressions for the slopes at the specular points for the quasi parallel and the quasi antiparallel cases (10) and (13). For backscatter at normal incidence the major contributions to the double scatter cross sections come from quasi horizontal paths between points 1 and 2 (see Fig. 1) and the slopes at the stationary points are approximately $\pm 45^\circ$.

II. NUMERICAL EXAMPLES

The incoherent double scatter cross sections (quasi parallel + quasi antiparallel) for two dimensional rough surfaces are plotted in Figs. 2, 3 and 4 as functions of the scatter angle $\vartheta^s \cos\varphi^s$ (where $\varphi^i = 0$, $\varphi^s = 0, \pi$). The two dimensional rough surface is assumed to be coated with gold of permittivity $\varepsilon_r = -9.888312 - j1.051766$ at $\lambda = 0.633\mu m$. The incident angle is equal to 10°. The Rayleigh roughness parameter $\beta = 4k_0^2\langle h^2\rangle$ is assumed to equal 394.105 in Figs. 2 and 4. In Figs. 2 and 3, the VV polarized double scatter cross sections are shown. The effect of changing the mean square slope (m.s.s.) is shown in Fig. 2 where it is assumed to be 0.25, 0.4, 0.5, 0.65, 0.75, 0.85 and 1.0. The level of the peak enhanced backscatter cross section is highest for mean square slope equal to 1.0 and it decreases as the mean square slope decreases to 0.25. In Fig. 3, the mean square slope is assumed to be 0.5 and the Rayleigh roughness parameter is assumed to be 10, 50, 100, 200, 300, and 394.105. The level of the peak enhanced backscatter increases as the Rayleigh roughness parameter increases. The width of the enhanced backscatter peak increases as both the Rayleigh roughness parameter and the mean square slope decrease. The cross sections for different polarizations (VV, HH, VH, HV) are shown in Fig. 4 in which the mean square slope is assumed to be 0.5 and β is assumed to be 394.105. Enhanced backscatter is observed for the four polarizations considered. The levels of the peak double scatter cross sections (in the backward direction) are approximately the same for both the like and cross polarized cases. The sharp backscatter enhancement observed for all polarizations, is associated with the quasi antiparallel double scatter paths.

CONCLUSIONS

The results for the double scatter radar cross sections exhibit sharp enhancement in the backscatter direction at normal and oblique incident angles. This sharp enhancement is associated with the quasi antiparallel double scatter path. The height and width of the peak in the backscatter direction depends on the mean square height and slope of the two dimensional random rough surface. The high frequency approximations make the computations more tractable; however, the polarization dependence is less obvious [3].

ACKNOWLEDGMENT

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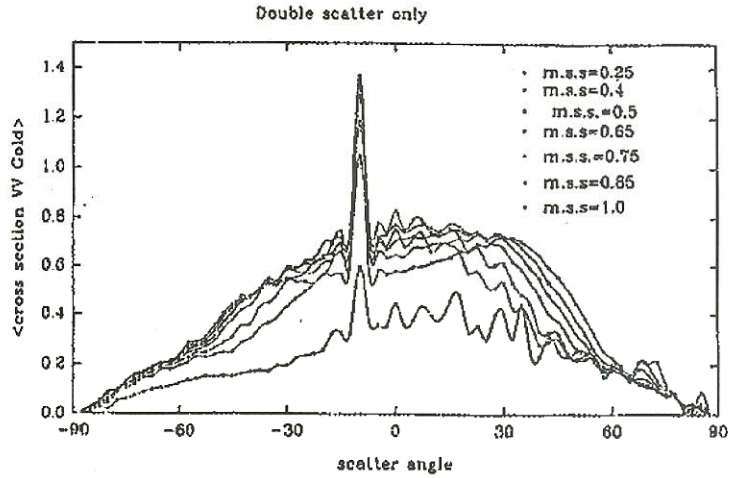

Fig. 2, ϑ^i=10°, ε_r=-9.888312-j1.051766, λ=0.633 μ.m., β=394.105.

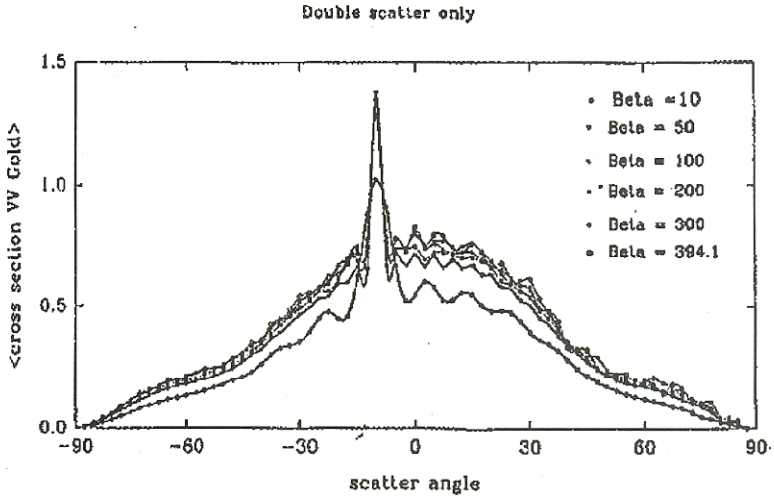

Fig. 3, ϑ^i=10°, ε_r=-9.888312-j1.051766, λ=0.633 μ.m., mean square slope=0.5.