

A Comparative Study of Different Tomography Methods for Breast Cancer Application

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Abstract-The solution of three dimensional inverse scattering problem using first-order Born and Rytov approximations are implemented for the reconstruction of an unknown dielectric sphere. Synthetic scattered field data from a sphere are generated using the Mie scattering method. The results show that the Rytov approximation provides better estimation of the sphere's parameters in comparison with the Born approximation.

I. INTRODUCTION

Three dimensional image reconstruction using microwave inverse scattering techniques has many applications. Those include nondestructive defectoscopy, ground penetrating radar (GPR) and several medical applications such as cancer and hypothermia detection. Focusing on medical applications, we find that three dimensional image reconstruction can be used to determine the electrical properties of biological tissue. The electrical properties are then utilized to determine whether a tissue is normal (healthy) or abnormal (malignant for example).

The subject of microwave imaging is to reconstruct the object by using the field scattered when the target is illuminated by electromagnetic waves. For reconstruction, the scattered field data are measured and processed. There are many techniques for microwave imaging but the practical applications of microwave imaging are not wide-spread because these methods tend to be computationally intensive and hence significantly time consuming. The reason behind such complexity is that, in the general case the inverse scattering problem is non-linear and ill-posed problem [1].

If the object to be reconstructed is a weak scatterer, the inverse problem can be linearized and solved using either the Born or Rytov approximations. In the Born approximation, one assumes that there is a linear relationship between the magnitude of scattered field and the scattering object while in the Rytov approximation, this linear relationship exists between the phase of scattered field and the scattering object. Born and Rytov approximations are common methods for solving the linearized inverse scattering problem. It is generally believed that the Rytov approximation, under certain conditions gives more accurate results than the Born approximation [2].

To our knowledge, most implementations of inverse scattering techniques in the literature are in two dimensions

(2D) since the solution of inverse problems in three-dimensions (3D), is much more complicated. Relying only on 2D algorithms causes the loss of critical information important for reconstructing a wide range of objects. When the Born and Rytov approximations are used for weakly scattering objects, the Fourier Transform of the scattered field data are in a line perpendicular to the incident wave direction in the 2D case and in a plane perpendicular to the incident wave direction in the 3D case. These transformed fields are proportional to the Fourier Transform of the object's shape (pixel based). After calculating the spatial frequencies, the scattering object can be reconstructed by applying the inverse Fourier Transform. This reconstruction process can be achieved using either the Direct Fourier Interpolation (DFI) or the Filtered Backpropagation (FBP) methods [3].

The difference between these approaches lies in the way in which the transformed scattered data is interpolated. In the Direct Fourier algorithm, the data are interpolated in the spatial frequency domain while in the Filtered Backpropagation algorithm, this interpolation is implemented directly in the space domain [3].

In this paper, the DFI method is employed in both the Born and Rytov approximation methods to reconstruct the permittivity of a dielectric sphere immersed in air. Advantages and disadvantages of each method are discussed.

II. FORMULATIONS

A. Scattering by sphere

When a plane electromagnetic wave illuminates a dielectric sphere that is surrounded by a non-magnetic and non-conducting medium, as illustrated in Figure 1, the exact scattered field data can be obtained using the Mie scattering method. Assume that the incident wave is propagating in the z -direction and its electric field is in the x -direction as $(E^i = \exp(ik_1 z) \hat{x})$. Also, assume that the sphere has a radius of a and the electromagnetic field within the sphere has a wave number k_1 , and in surrounding medium it has a wave number k_2 .

In this work, we consider the scattered fields in the far field zone. With this assumption we have these relationships for scattered electric field [4, 5]:

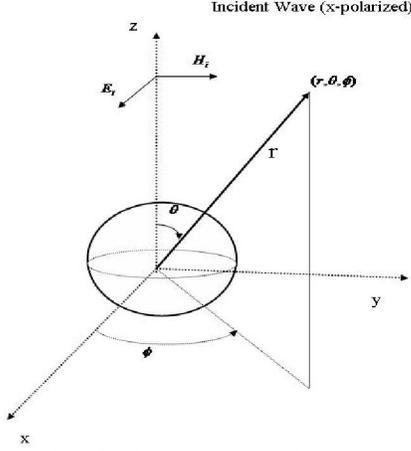


Fig. 1: Scattering of a plane electromagnetic wave by a sphere

$$E_s^\phi = \frac{-i \exp(-ik_2 r)}{k_2 r} \sin \phi \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left\{ a_n \frac{P_n^{(1)}(\cos \theta)}{\sin \theta} + b_n \frac{dP_n^{(1)}(\cos \theta)}{d\theta} \right\} \quad (1)$$

$$E_s^\theta = \frac{i \exp(-ik_2 r)}{k_2 r} \cos \phi \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left\{ a_n \frac{dP_n^{(1)}(\cos \theta)}{d\theta} + b_n \frac{P_n^{(1)}(\cos \theta)}{\sin \theta} \right\} \quad (2)$$

Here E_ϕ is the ϕ -component and E_θ is the θ -component of the scattered electric field. Where $k_2 = 2\pi m_2 / \lambda_0$ and λ_0 is the wavelength in free space and coefficients of a_n and b_n are given by [4, 5]:

$$a_n = \frac{\psi_n(\alpha) \psi_n'(\beta) - m \psi_n(\beta) \psi_n'(\alpha)}{\zeta_n(\alpha) \psi_n'(\beta) - m \psi_n(\beta) \zeta_n'(\alpha)} \quad (3)$$

$$b_n = \frac{m \psi_n(\alpha) \psi_n'(\beta) - \psi_n(\beta) \psi_n'(\alpha)}{m \zeta_n(\alpha) \psi_n'(\beta) - \psi_n(\beta) \zeta_n'(\alpha)} \quad (4)$$

where $\alpha = 2\pi m_2 a$, $\beta = 2\pi m_1 a$, and a is the radius of the sphere, m_1 is the refractive index of the sphere and m_2 is the refractive index of medium with $m = m_1 / m_2$. Also $\psi_n(\alpha) = \alpha j_n(\alpha)$, $\psi_n(\beta) = \beta j_n(\beta)$, $\zeta_n(\alpha) = \alpha h_n^{(2)}(\alpha)$. The spherical Bessel function is given by j_n , and $h_n^{(2)}$ is the spherical Hankel function. $\psi_n'(\alpha)$ is the derivative of ψ_n with respect to its argument and $\zeta_n'(\alpha)$ is the derivative of ζ_n with respect to its argument. The scattered electric field in the reconstruction algorithm is $E_x^s = \cos \theta \cos \phi E_\theta^s - \sin \phi E_\phi^s$ since the incident plane wave is polarized in x-direction. We use the results of scattered field as synthetic data for reconstruction algorithm.

B. The Direct Fourier Interpolation Algorithm

The interaction of an illuminating wave field $\psi_{inc}(\vec{r})$ with a scattering object in the frequency domain can be described by the Lippmann-Schwinger equation [6]:

$$\psi(\vec{r}) = \psi_{inc}(\vec{r}) + \iiint_V g(\vec{r} - \vec{r}') o(\vec{r}') \psi(\vec{r}') d^3 \vec{r}' \quad (5)$$

where $g(\vec{r} - \vec{r}')$ is the three-dimensional Green's function of the background, $o(\vec{r})$ is the object function to be reconstructed, $\psi(\vec{r})$ is the total field (the sum of incident field and scattered field) inside or outside of the scattering object, and V is a volume that surrounds the scattering object.

The object function is related to constitutive parameters of the scattering object by

$$o(\vec{r}) = k_0^2 (n(\vec{r})^2 - 1) \quad (6)$$

where $n(\vec{r})$ is the complex refractive index of the scattering object and $k_0 = \frac{2\pi}{\lambda}$ (λ is the propagation wavelength in the surrounding medium). The Green function in the 3-D case is

$$g(\vec{r} - \vec{r}') = \frac{e^{jk_0 |\vec{r} - \vec{r}'|}}{|\vec{r} - \vec{r}'|} \quad (7)$$

Assuming that the incident field is a plane wave propagating in +z-direction $\psi_{inc} = e^{-jk_0 z}$, the Lippmann-Schwinger equation can be written under the first-order Born approximation as:

$$\psi_{sc}(\vec{r}) = \iiint_V g(\vec{r} - \vec{r}') o(\vec{r}') e^{-jk_0 z'} d^3 \vec{r}' \quad (8)$$

where the scattered field is $\psi_{sc}(\vec{r}) = \psi(\vec{r}) - \psi_{inc}(\vec{r})$ and the 3-D Green's function can be expanded in following way [6]:

$$g(\vec{r} - \vec{r}') = \frac{j}{8\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{\gamma} e^{j[\alpha(x-x') + \beta(y-y') + \gamma(z-z')]} d\alpha d\beta \quad (9)$$

$$\gamma = \sqrt{k_0^2 - \alpha^2 - \beta^2} \quad (10)$$

Substituting (9) into (8) and after some mathematical manipulations yields:

$$\tilde{O}(\omega_x, \omega_y, k_0 + \sqrt{k_0^2 - \omega_x^2 - \omega_y^2}) = \quad (11)$$

$$-2j \sqrt{k_0^2 - \omega_x^2 - \omega_y^2} \exp(-j \sqrt{k_0^2 - \omega_x^2 - \omega_y^2} l_0) \tilde{\psi}_{sc, z=l_0}(\omega_x, \omega_y)$$

$$\tilde{\psi}_{sc, z=l_0}(\omega_x, \omega_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \psi_{sc}(x, y, z=l_0) e^{-j(x\omega_x + y\omega_y)} dx dy \quad (12)$$

$$\psi_{sc}(x, y, z=l_0) = E_x^{scat}(x, y, z=l_0) \quad (13)$$

Where $\tilde{\psi}_{sc, z=l_0}(\omega_x, \omega_y)$ is the Fourier transform of the

scattered field in plane $z = l_0$ perpendicular to incident direction.

Equation (11) indicates how the Fourier transform of the measured scattered field data in a plane perpendicular to the propagation direction of the incident field is proportional to the Fourier transform of the object function on the surface of a so-called Ewald sphere centered at $(0, 0, k_0)$ with radius k_0 .

When the Rytov approximation is used, the integral equation for the complex phase of the scattered field is [6,7]:

$$P_s(\vec{r}) = \frac{1}{\psi_{inc}(\vec{r})} \int_{V'} g(\vec{r} - \vec{r}') \psi_{inc}(\vec{r}') o(\vec{r}') d^3 r' \quad (14)$$

Following a procedure similar to the one used for Born Approximation, we obtain a similar expression for Rytov approximation:

$$\begin{aligned} \tilde{O}(\omega_x, \omega_y, k_0 + \sqrt{k_0^2 - \omega_x^2 - \omega_y^2}) = \\ \frac{-2j\sqrt{k_0^2 - \omega_x^2 - \omega_y^2} \exp(-j\sqrt{k_0^2 - \omega_x^2 - \omega_y^2} l_0)}{\exp(-jk_0 l_0)} \tilde{P}_{s,z=l_0}(\omega_x, \omega_y) \end{aligned} \quad (15)$$

where,

$$\tilde{P}_{s,z=l_0}(\omega_x, \omega_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} P_s(x, y, z = l_0) e^{-j(x\omega_x + y\omega_y)} dx dy \quad (16)$$

is the Fourier transform of the complex phase of the scattered field at $z = l_0$. The complex phase can be obtained with the following relationship:

$$P_s(x, y, z = l_0) = \ln \left(1 + \frac{E_x^{scat}(x, y, z = l_0)}{E_x^{inc}(x, y, z = l_0)} \right) \quad (17)$$

C. Discretization of Problem

For reconstruction, the scattered field data in a plane normal to the incident direction is used. Assume that M is the number of point receivers in the x-direction and N is the number of point receivers in the y-direction. Hence, the discretization steps in the x and y directions are:

$$\Delta x = \frac{X_{\max} - X_{\min}}{M - 1} \quad \Delta y = \frac{Y_{\max} - Y_{\min}}{N - 1} \quad (18)$$

where X_{\min} , X_{\max} , Y_{\min} and Y_{\max} are the limits of rectangular domain in space domain.

The Fourier transform of the object can be discretized as

$$\tilde{O}_{2D}[q, r] = \tilde{O}_{2D}(q \Delta \alpha, r \Delta s) = -2j\sqrt{k_0^2 - (q \Delta \alpha)^2 - (r \Delta s)^2} \times$$

$$\exp(-j\sqrt{k_0^2 - (q \Delta \alpha)^2 - (r \Delta s)^2} l_0) \times$$

$$\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} E_{x,scat}[m, n] e^{-j(mq \Delta \alpha \Delta x + nr \Delta \beta \Delta y)} \quad (19)$$

where,

$$E_{x,scat}[m, n] = E_x^{scat}(x = m \Delta x, y = n \Delta y) \quad (20)$$

We select $\Delta \alpha$ and Δs (discretization steps in Fourier domain) according to the condition $(q \Delta \alpha)^2 + (r \Delta s)^2 < k_0^2$ as:

$$\Delta \alpha = \frac{\sqrt{2} k_0}{2Q} \quad \Delta s = \frac{\sqrt{2} k_0}{2R} \quad (21)$$

Where $-Q \leq q \leq Q$ and $-R \leq r \leq R$. The total number of samples in the Fourier domain is $(2Q+1) \times (2R+1)$. The object can be obtained with the following relationship:

$$o_{2D}(x, y) = \frac{1}{(2\pi)^2} \sum_{q=-Q}^Q \sum_{r=-R}^R \tilde{O}_{2D}[q, r] e^{-j(mq \Delta \alpha \Delta x + nr \Delta \beta \Delta y)} \quad (22)$$

III. DISCUSSION AND RESULTS

Synthetic scattered field data were generated using the Mie scattering method. The measurement planes are defined to be 25λ away from the origin. A dielectric sphere with a dielectric constant of $\epsilon_r = 1.5$ and radius 5λ was investigated. The frequency of incident wave is selected to be 1 GHz. The total number of point receivers in x- and y-direction is 10000 (100 points in each direction), with spacing of $\Delta x = \frac{\lambda}{4}$, $\Delta y = \frac{\lambda}{4}$. The number of samples in the Fourier domain is 401×401 .

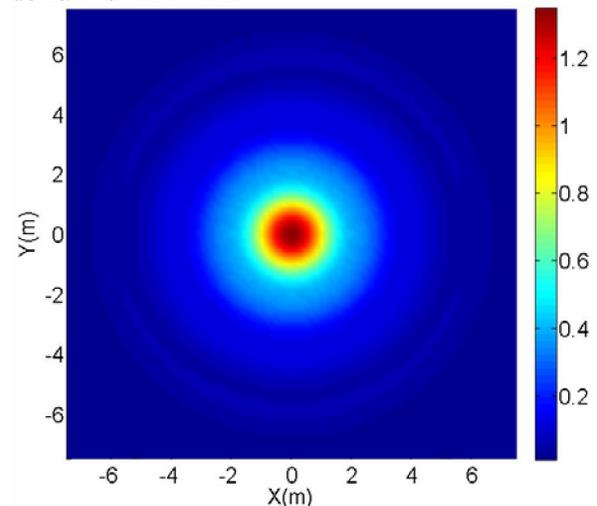


Fig 2: Reconstructed Permittivity of a sphere using Born Approximation

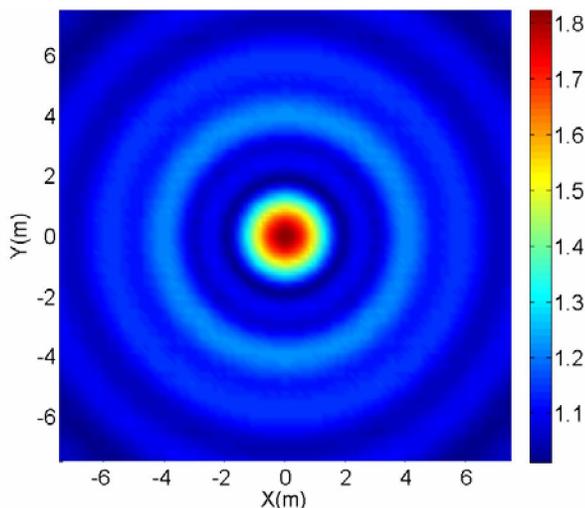


Fig 3: Reconstructed Permittivity of a sphere using Rytov Approximation

The reconstructed image using Born approximation is shown in Figure 2 and the reconstructed image using Rytov Approximation is shown in Figure 3. The results show that the Born approximation has better prediction of the sphere's permittivity but the Rytov approximation can reconstruct the sphere's radius more precisely.

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